Quantum Gravity

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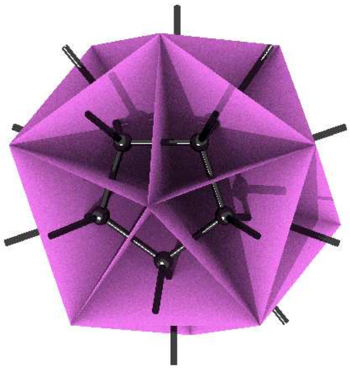

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Contents

I Relativistic foundations 1

Preface

A dream I have long held was to write a "treatise" on quantum gravity once the theory had been finally found and experimentally confirmed. We are not yet there. There isn't either experimental support nor large theoretical consensus. Still, a large amount of work has been developed over the last twenty years towards a quantum theory of spacetime. Many issues have been clarified, and a definite approach has crystallized. The approach, variously denoted[1], is mostly known as "loop quantum gravity".

The problem of quantum gravity has many aspects. Ideas and results are scattered in the literature. In this book I have attempted to collect the main results and to present an overall perspective on quantum gravity, as developed during this period. The point of view is personal and the choice of subjects is determined by my own interests. I apologize with friends and colleagues for what is missing: the reason so much is missing is in my own limits, for which I am the first to be sorry.

It is difficult to underestimate the vastitude of the problem of quantum gravity. The physics of the early XXth century has modified our understanding of the physical world in depth, changing the meaning of the basic concepts we use to grasp it: matter, causality, space and time. We haven't found a consistent picture of the world in which these modifications make sense together, yet. The problem of quantum gravity is nothing less than the problem of finding the novel consistent picture, finally bringing the XXth century scientific revolution to an end.

Solving a problem of this sort is not just a matter of mathematical skill. Like for the birth of quantum mechanics, relativity, electromagnetism, and newtonian mechanics, there are conceptual and foundational problems to be addressed. We have to understand which (possibly new) notions make sense and which old notions must be discarded, in order to describe spacetime in the quantum relativistic regime. What we need is not just a technique for computing, say, graviton-graviton scattering amplitudes (although we certainly want to be able to do so, eventually). What we need is to understand how to think the world at the light of what we have learned about it with quantum theory and general relativity.

General relativity, in particular, has modified our understanding of the spatio-temporal structure of reality in a way whose consequences have not been fully explored yet. A consistent part of the research in quantum gravity explores foundational issues, and Part I of this book ("Relativistic foundations") is devoted to basic issues. It is an exploration on how to rethink basic physics from scratch, after the general-relativistic conceptual revolution. Without this, we risk to ask any tentative quantum theory of gravity the wrong kind of questions.

Part II of the book ("Loop quantum gravity") focuses on the loop approach. The loop theory, described in Part II, can be studied by itself, but its reason and interpretation are only clear in the light of the general framework studied in Part I. Although several aspects of this theory are still incomplete, the subject is mature for a book. A theory begins to be credible only when its

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[1] See the notation section.

original predictions are reasonably unique and are confirmed by new experiments. Loop quantum gravity is not yet credible in this sense. Nor is any other current tentative theory of quantum gravity. The interest of the loop theory, in my opinion, is that at present it is the only approach to quantum gravity leading to well defined physical predictions (falsifiable, at least in principle) and, more importantly, it is the most determined effort for a genuine merge of quantum field theory with the world view that we have discovered with general relativity. The future will tell us more.

There are several other introductions to loop quantum gravity. Classic reports on the subject, illustrating various stages of the development of the theory are, in chronological order, [1, 2, 3, 4, 5, 6, 7, 8]. For a rapid orientation, and to appreciate different points of view, see the review papers [9, 10, 11, 12, 13]. Much useful material can be found in [14]. Good introductions to spin foam theory are in [9, 15, 16, 17]. This book is self contained, but I have tried to avoid excessive duplications, referring to other books and review papers for non-essential topics well developed elsewhere. This book focuses on physical and conceptual aspects of loop quantum gravity. Thomas Thiemann's book [18], which is going to be completed soon, focuses on the mathematical foundation of the same theory. The two books are complementary and can almost be read as Volume 1 ("Introduction and conceptual framework") and Volume 2 ("Complete mathematical framework") of a general presentation of loop quantum gravity.

The book assumes that the reader has a basic knowledge of general relativity, quantum mechanics and quantum field theory. In particular, the aim of the chapters on general relativity (chapter 2), classical mechanics (ch 3), hamiltonian general relativity (ch 4), and quantum theory (ch 5) is to offer the fresh perspective on these topics which is needed for quantum gravity, to a reader that already knows the conventional formulation of these theories.

Sections with comments and examples are printed in smaller fonts. Sections that contain side or more complex topics and that can be skipped in a first reading without compromising the understanding of what follows are marked with a star ($*$) in the title. References in the text appear only when strictly needed for comprehension. Each chapter ends with a short bibliographical section, pointing out essential references for the reader who wants to go more in detail or to trace original works on a subject. I have given up the immense task of collecting a full bibliography on loop quantum gravity. On many topics I refer to specific review articles where ample bibliographic information can be found. An extensive bibliography on loop quantum gravity is in [18].

I have written this book thinking of a researcher interested in working in quantum gravity, but also of a good PhD student or an open minded scholar, curious about this extraordinary open problem. I have found the journey towards general relativistic quantum physics, towards quantum spacetime, a fascinating adventure. I hope the reader will see the beauty I see, and that he or she will be capable to complete the journey. The landscape is magic, the trip is far from being over.

Marseille, Toronto, Rome, 2002-2003

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With all these friends I have had the joy of talking about a physics which is far from problem-solving, from outsmarting each other, or from making weapons to make us stronger than them. I think that physics is about escaping the prison of the received thoughts and searching for novel ways of thinking the world, about trying to clear a bit the misty lake of our insubstantial dreams, which reflect reality like the lake reflects the mountains.

Foremost, thanks to Bonnie, she knows why.

Terminology and notation

In the book, "relativistic" means "*general* relativistic", unless otherwise specified. When referring to *special* relativity, I say so explicitly. Similarly, "nonrelativistic" and "prerelativistic" mean "non *general* relativistic" and "pre-*general*-relativistic". The choice is a bit unusual (special relativity, in this language, is "nonrelativistic"). One reason for it is simply to make language smoother: the book is about *general* relativistic physics, and repeating "*general*" every other line sounds too much like a Frenchman talking about de Gaulle. But there is a more substantial reason: the complete revolution in spacetime physics, which truly deserves the name of relativity is the one of general relativity, not the one of special relativity. This opinion is not always shared today, but it was Einstein's opinion. Einstein has been criticized on this; but in my opinion the criticisms miss the full reach of Einstein's discovery about spacetime. One of the aims of this book is to defend in modern language Einstein's intuition that his gravitational theory is the full implementation of relativity in physics. This point is discussed at length in chapter 2.

I often indulge to the bad physicists' habit of mixing up function (f) and function values ($f(x)$). Care is used when relevant. Similarly, I follow standard physicists abuse of language in denoting a field such as the Maxwell potential as $A_\mu(x)$, $A(x)$, or A, where the three notations are treated as equivalent manners of denoting the field. Again, care is used where relevant.

All fields are assumed to be smooth, unless otherwise specified. All statements about manifolds and functions are local unless otherwise specified; that is, they hold within a single coordinate patch. In general I do not specify the domain of definition of functions: clearly equations hold where functions are defined.

Index notation follows the most common choice in the field: Greek indices from the middle of the alphabet $\mu, \nu, \ldots = 0, 1, 2, 3$ are 4d spacetime tangent indices. Capital Latin indices from the middle of the alphabet $I, J, \ldots = 0, 1, 2, 3$ are 4d Lorentz tangent indices. (In the special relativistic context the two are used without distinction.) Lowercase Latin indices from the beginning of the alphabet $a, b, \ldots = 1, 2, 3$ are 3d tangent indices. Lowercase Latin indices from the middle of the alphabet $i, j, \ldots = 0, 1, 2, 3$ are 3d indices in R^3. Coordinates of a 4d manifold are usually indicated as $x, y \ldots$, while 3d manifold coordinates are usually indicated as $\vec{x}, \vec{y}$ (also as $\vec{r}$). Thus the components of a spacetime coordinate x are

$$x^\mu = (t, \vec{x}) = (x^0, x^a); \tag{1}$$

while the components of a Lorentz vector e are

$$e^I = (e^0, e^i). \tag{2}$$

η_{IJ} is the Minkowski metric, with signature $[-,+,+,+]$. The indices $I, J \ldots$ are raised and lowered with η_{IJ}. δ_{ij} is the Kronecker delta, or the R^3 metric. The indices $i, j \ldots$ are raised and lowered with δ_{ij}.

For reasons explained at the beginning of chapter 2, I call "gravitational field" the tetrad field $e^I_\mu(x)$, instead of the metric tensor $g_{\mu\nu}(x) = \eta_{IJ}\; e^I_\mu(x) e^J_\nu(x)$.

ϵ_{IJKL}, or $\epsilon_{\mu\nu\rho\sigma}$, is the completely antisymmetric object with $\epsilon_{0123} = 1$. Same for ϵ_{abc}, or ϵ_{ijk}, in 3d. The Hodge star is defined by

$$F^*_{IJ} = \epsilon_{IJKL} F^{KL} \tag{3}$$

in flat space, and by the same equation, where $F_{IJ} e^I_\mu e^J_\nu = F_{\mu\nu}$ and $F^*_{IJ} e^I_\mu e^J_\nu = F^*_{\mu\nu}$ in the presence of gravity. Equivalently,

$$F^*_{\mu\nu} = \sqrt{-\det g}\; \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma} = |\det e|\; \epsilon_{\mu\nu\rho\sigma} F^{\rho\sigma}. \tag{4}$$

Symmetrization and antisymmetrization of indices is defined with a half: $A_{(ab)} = \frac{1}{2}(A_{ab} + A_{ba})$ and $A_{[ab]} = \frac{1}{2}(A_{ab} - A_{ba})$.

I call "curve" on a manifold M, a map

$$\begin{aligned} \gamma : I \;&\to\; M \\ s \;&\mapsto\; \gamma^a(s), \end{aligned} \tag{5}$$

where I is an interval of the real line R (possibly the entire R.) I call "path" an oriented unparametrized curve, namely an equivalence class of curves under change of parametrization $\gamma^a(s) \mapsto \gamma'^a(s) = \gamma^a(s'(s))$, with $ds'/ds > 0$.

An orthonormal basis in the Lie algebras $su(2)$ and $so(3)$ is chosen once and for all and these algebras are identified with R^3. For $so(3)$, the basis vectors $(v_i)^j{}_k$ can be taken proportional to $\epsilon_i{}^j{}_k$; for $su(2)$, the basis vectors $(v_i)^A{}_B$ can be taken proportional to the Pauli matrices, recalled in the appendix. Thus, an algebra element ω in $su(2) \sim so(3)$ has components ω^i.

For any antisymmetric quantity v^{ij} with two 3d indices i, j, I use also the one-index notation

$$v^i = \tfrac{1}{2}\,\epsilon^i{}_{jk}\; v^{jk}, \qquad\qquad v^{ij} = \epsilon^{ij}{}_k\; v^k; \tag{6}$$

the one-index and the two-indices notation are considered defining the same object. For instance the $SO(3)$ connections ω^{ij} and A^{ij}, are equivalently denoted ω^i and A^i.

Symbols. Here is a list of symbols, with their name and the equation or the section where they are introduced or defined.

A	area	Sec 2.1.4
A	Yang-Mills connection	Eq 2.27
$A, A^i_\mu(x)$	selfdual 4d gravitational connection	Eq 2.16
$A, A^i_a(\vec{x})$	selfdual or real 3d gravitat. conn.	Sec 4.1.1, 4.2
$\mathcal{C}$	relativistic configuration space	Sec 3.2.1
D_μ	covariant derivative	Eq 2.28
Diff*	extended diffeomorphism group	Sec 6.2.2
$e^I_\mu(x)$	gravitational field	Eq 2.1
e	determinant of e^I_μ	
e	edge (of spinfoam)	Sec 9.1
$E, E^a_i(\vec{x})$	gravitational electric field	Sec 4.1.1

f	face (of spinfoam)	Sec 9.1
F	curvature two-form	Sec 2.1.1
g or U	group element	
G	Newton constant	
$\mathcal{G}$	space of boundary data	Sec 3.2.5-3.3.3
h_γ	$U(A,\gamma)$	Sec 7.1
H	relativistic hamiltonian	Sec 3.2
H_0	nonrelativistic (conventional) hamilt	Sec 3.2
$\mathcal{H}$	quantum state space	Ch 5
$\mathcal{H}_0$	nonrelativistic quantum state space	Ch 5
i_n	intertwiner on spinnetwork node n	Sec 6.3
i_e	intertwiner on spinfoam edge e	Ch 9
j	irreducible rep (for $SU(2)$: spin)	
j_l	spin associated to spinnetwork link l	Sec 6.2.1
j_f	rep associated to spinfoam face f	Ch 9
$\mathcal{K}$	kinematical quantum state space	Sec 5.2
$\mathcal{K}_0$	$SU(2)$ invariant quantum state space	Sec 6.2.3
$\mathcal{K}_{\rm Diff}$	diff invariant quantum state space	Sec 6.2.3
K	boundary quantum space	Sec 5.1.4 5.3.5
l	link (of spin network)	Sec 9.1
$\mathbf{L}$	length	Sec 2.1.4
M	spacetime manifold	
n	node (of spin network)	Sec 9.1
p_a	relativistic momenta (including p_t)	Sec 3.2
p_t	momentum conjugate to t	Sec 3.2
P	the "projector" operator	Sec 5.2
P_{G}	group G projector	Eq 9.117
P_{H}	subgroup H projector	Eq 9.119
$\mathcal{P}$	transition probability	Ch 5
q^a	partial observables	Sec 3.2
$R^I{}_{J\,\mu\nu}(x)$	curvature	Eq 2.8
$R^{(j)\alpha}{}_\beta(g)$	matrix of group element g in repr j	
$\mathcal{R}$	3d region	Sec 2.1.4
s	s-knot : abstract spin network	Eq 6.4.1
$\|s\rangle$	s-knot state	Eq 6.4.1
S_{BH}	black hole entropy	Sec 8.2
S	imbededd spin network	Sec 6.3
$\|\mathrm{S}\rangle$	spin network state	Sec 6.3.1
$\mathcal{S}$	2d surface	Sec 2.1.4
$\mathcal{S}$	space of fast decrease functions	Ch 5
$\mathcal{S}_0$	space of tempered distributions	Ch 5
$S[\tilde{\gamma}]$	action functional	Sec 3.2
$S(q^a)$	Hamilton-Jacobi function	Sec 3.2.2
$S(q^a,q^a_0)$	Hamilton function	Sec 3.2.5
t_ρ	thermal time	Sec 3.4 and 5.5.1
T	target space of a field theory	Sec 3.3.1
U or g	group element	
$U(A,\gamma)$	holonomy	Sec 2.1.5
v	vertex (of spinfoam)	Sec 9.1
$\mathbf{V}$	volume	Sec 2.1.4

$W(q^a, q'^a)$	propagator	Ch 5
W	transition amplitudes, propagator	Sec 5.2
x	4d spacetime coordinates	
$\vec{x}$	3d coordinates	
Z	partition function	Ch 9
α	loop, closed path	
β	inverse temperature	Sec 3.4
γ	path	
γ	motion (in $\mathcal{C}$)	Sec 3.2.1
γ	Immirzi parameter	Sec 4.2.3
$\tilde{\gamma}$	motion in Ω	Sec 3.2
Γ	relativistic phase space	Sec 3.2.1
Γ	graph	Ch 6.2
Γ	two-complex	Ch 9
θ	Poincaré-Cartan form on Σ	Sec 3.2.2
$\tilde{\theta}$	Poincaré form on Ω	Eq 3.9
$\eta_{\mu\nu}$	Minkowski metric = diag[-1,1,1,1]	
λ	cosmological constant	Eq 2.9
λ	gauge parameter	Sec 2.1.3
ρ	statistical state	Sec 3.4 and 5.5.1
Σ	constraint surface $H = 0$	Sec 3.2.2
Σ	3d boundary surface	Ch 4
σ	spinfoam	Ch 9
$\phi(x)$	scalar field	Eq 2.29
$\psi(x)$	fermion field	Eq 2.32
ω	presymplectic form on Σ	Sec 3.2.2
$\omega^I_{\mu J}(x)$	spin connection	Eq 2.2
$\tilde{\omega}$	symplectic form on Ω	Sec 3.2.2
Ω	space of observables and momenta	Sec 3.2-3.3.2
$\{6j\}$	Wigner 6j symbol	Eq 9.33
$\{10j\}$	Wigner 10j symbol	Eq 9.103
$\{15j\}$	Wigner 15j symbol	Eq 9.56
$\|0\rangle$	covariant vacuum in K	Sec 5.1.4, 5.3.5
$\|0_t\rangle$	dynamical vacuum in $\mathcal{K}_t$	Sec 5.1.4, 5.3.2
$\|0_M\rangle$	Minkowski vacuum in $\mathcal{H}$	Sec 5.1.4, 5.3.1

The name of the theory. Finally, a word about the name of the quantum theory of gravity described in this book. The theory is known as *"loop quantum gravity"* (LQG), or sometimes *"loop gravity"* for short. However, the theory is also designated in the literature using a variety of other names. I list here these other names, and the variations of their use, for the benefit of the disoriented reader.

- *"Quantum Spin Dynamics"* (QSD) is used as a synonymous of LQG. Within LQG, it is sometimes used to designate in particular the dynamical aspects of the hamiltonian theory.

- *"Quantum geometry"* is sometimes used as a synonymous of LQG. Within the theory, it is used to designate in particular the kinematical aspects of the theory. The expression *"quantum geometry"* is generic: it is also widely used in other approaches to quantum spacetime, in particular, dynamical triangulations [19] and noncommutative geometry. Therefore it is not a good designation for a specific theory of quantum gravity.

- *"Nonperturbative quantum gravity"*, *"canonical quantum gravity"* and *"quantum general relativity"* (QGR) are often used to designate LQG, although their proper meaning is wider.

- The expression *"Ashtekar approach"* was used in the past to designate LQG: it comes from the fact that a key ingredient of LQG is the reformulation of classical GR as a theory of connections, developed in particular by Abhay Ashtekar.

- In the past, LQG was also called *"the loop representation of quantum general relativity"*. Today, *"loop representation"* and *"connection representation"* are used within LQG to designate representation of the states of LQG as functionals of loops (or spin networks) and, respectively, functionals of the connection. The two are related in the same manner as the energy ($\psi_n = \langle n|\psi\rangle$) and position ($\psi(x) = \langle x|\psi\rangle$) representations of the harmonic oscillator states.

Part I

Relativistic foundations

I know that I am mortal, and the creature of a day... but when I search out the massed wheeling circles of the stars, my feet no longer touch the earth: side by side with Zeus himself, I drink my fill of ambrosia, food of the gods...

Claudius Ptolemy, “Mathematical Syntaxis”

Chapter 1

General ideas and heuristic picture

The aim of this chapter is to introduce the general ideas on which this book is based and to present the picture of quantum spacetime that emerges from loop quantum gravity, in a heuristic and intuitive manner. The style of the chapter is therefore conversational, with little regard for precision and completeness. In the course of the book the ideas and notions introduced here will be made precise, and the claims will be justified and formally derived.

1.1 The problem of quantum gravity

1.1.1 Unfinished revolution

Quantum mechanics (QM) and general relativity (GR) have extended our understanding of the physical world widely. A large part of the physics of the last century has been a triumphant march of exploration of new worlds opened by these two theories. QM led to atomic physics, nuclear physics, particle physics, condensed matter physics, semiconductors, lasers, computers, quantum optics ... GR led to relativistic astrophysics, cosmology, GPS technology ...and is today leading us, hopefully, towards gravitational wave astronomy.

But QM and GR have destroyed the coherent picture of the world provided by prerelativistic classical physics: each was formulated in terms of assumptions contradicted by the other theory. QM was formulated using an external time variable (the t of the Schrödinger equation) or a fixed, nondynamical background spacetime (the spacetime on which quantum field theory is defined). But this external time variable and this fixed background spacetime are incompatible with GR. In turn, GR was formulated in terms of Riemannian geometry, assuming that the metric is a smooth and deterministic dynamical field. But QM requires that any dynamical field is quantized: at small scales it manifests itself in discrete quanta and is governed by probabilistic laws.

We have learned from GR that spacetime is dynamical and we have learned from QM that any dynamical entity is made by quanta and can be in probabilistic superposition states. Therefore at small scales there should be quanta of space and quanta of time, and quantum superposition of spaces. But what does this mean? We live in a spacetime with quantum properties: a *quantum spacetime.* What is quantum spacetime? How can we describe it?

Classical prerelativistic physics provided a coherent picture of the physical world. This was based on clear notions such as *time, space, matter, particles, waves, forces, measurements, deterministic laws...* This picture has partially evolved (in particular with the advent of field theory and special relativity) but it has remained consistent and quite stable for three centuries. GR and QM have then modified the basic notions in depth. GR has modified the notions of space and time; QM the notions of causality, matter, and measurements. The novel, modified notions do not fit together easily. The new coherent picture is not yet available. With all their immense empirical success, GR and QM have left us with an understanding of the physical world which is unclear and badly fragmented. At the foundations of physics there is today confusion and incoherence.

We want to combine what we have learnt about our world from the two theories and to find a new synthesis. This is a major challenge –perhaps the major challenge– in today's fundamental physics. GR and QM have opened a revolution. The revolution is not yet complete.

With notable exceptions (Dirac, Feynman, Weinberg, DeWitt, Wheeler, Penrose, Hawking, t'Hooft, among others) most of the physicists of the second half of last century have ignored this challenge. The urgency was to apply the two theories to larger and larger domains. The developments were momentous and the dominant attitude was pragmatic. Applying the new theories was more important than understanding them. But an overly pragmatic attitude is not productive in the long run. Towards the end of the XXth century, the attention of theoretical physics has been increasingly focusing on the problem of merging the conceptual novelties of QM and GR.

This book is the account of an efforts to do so.

1.1.2 How to search for quantum gravity?

How to search for this new synthesis? Conventional field quantization methods are based on the weak field perturbation expansion. Their application to GR fails because it yields a nonrenormalizable theory. Perhaps this is not surprising: GR has changed the notions of space and time too radically, to docilely agree with flat space quantum field theory. Something else is needed.

In science there are no secure recipes for discovery and it is important to explore different directions at the same time. Currently, a quantum theory of gravity is sought along various paths. The two most developed are loop quantum gravity, described in this book, and string theory. Other research directions include dynamical triangulations, noncommutative geometry, Hartle's quantum mechanics of spacetime (this is not really a specific quantum theory of gravity, but rather a general theoretical framework for general relativistic quantum theory), Hawking's euclidean sum over geometries, quantum Regge calculus, Penrose's twistor theory, Sorking's causal sets, t'Hooft deterministic approach and Finkelstein's theory. The reader can find ample references in the general introductions to quantum gravity mentioned in the note at the end of this chapter. Here, I sketch only the general ideas that motivate the approach described in this book, plus a brief comment on string theory, which is the most popular alternative to loop gravity.

Our present knowledge of the basic structure of the physical universe is summarized by GR, quantum theory and quantum field theory (QFT), and the particle physics standard model. This set of fundamental theories is inconsistent. But it is characterized by an extraordinary empirical success, nearly unique in the history of science. Indeed, currently there is no evidence of any observed phenomenon that clearly escapes, questions or contradicts this set of theories (or a minor modification of the same, to account, say, for a neutrino mass or a cosmological constant). This set of theories becomes meaningless in certain physical regimes. In these regimes, we expect the predictions of quantum gravity to become relevant and to differ from the predictions of GR and the standard model. But these regimes are outside our experimental or observational reach, at least so far. Therefore, we have no direct empirical guidance for searching for quantum gravity – as, say, atomic spectra guided the discovery of quantum theory.

Since quantum gravity is a theory expected to describe regimes that are, so far, inaccessible, one might worry that anything could happen in these regimes, at scales far removed from our experience. Maybe the search is impossible because the space of the possible theories is too large. This worry is unjustified. If this was the problem, we would have plenty of complete, predictive and coherent theories of quantum gravity. Instead, the situation is precisely the opposite: we haven't any. The fact is that we do have plenty of information about quantum gravity, because we have QM and we have GR. Consistency with QM and GR is an extremely strict constraint.

A view is sometime expressed that some totally new, radical and wild hypothesis is needed for quantum gravity. I do not think that this is the case. Wild ideas pulled out of the blue sky have never made science advance. The radical hypotheses that physics has successfully adopted have always been reluctantly adopted because they were forced by new empirical data –Kepler's ellipses, Bohr's quantization ... – or by stringent theoretical deductions –Maxwell inductive current, Einstein's relativity... (See appendix C). Generally, arbitrary novel hypotheses lead nowhere.

In fact, today we are precisely in one of the typical situations in which theoretical physics has worked at its best in the past. Many of the most striking advances in theoretical physics have derived from the effort of finding a common theoretical framework for two basic and apparently conflicting discoveries. For instance, the aim of combining special relativity and non relativistic quantum theory led to the theoretical discovery of antiparticles; combining special relativity with Newtonian gravity led to general relativity; combining the Keplerian orbits with Galilean physics led to Newton's mechanics; combining Maxwell theory with Galilean relativity led to special relativity, and so on. In all these cases, major advances have been obtained by "taking seriously"[1] apparently conflicting theories, and exploring the implications of holding the key tenets of both theories for true. Today we are precisely in one of these characteristic situations. We have learned two new very general "facts" about nature, expressed by QM and GR: we have "just" to figure out what they imply, taken together. Therefore, the question we have to ask is: what have we really learned about the world from QM and from GR? Can we combine these insights into a coherent picture? What we need is a conceptual scheme in which the insights obtained with GR and QM fit together.

This view is *not* the majority view in theoretical physics, at present. There is consensus that QM has been a conceptual revolution, but many do not view GR in the same way. According to many, the discovery of GR has been just the writing of one more field theory. This field theory is, furthermore, likely to be only an approximation to a theory we do not yet know. According to this opinion, GR should not be taken too seriously as a guidance for theoretical developments.

I think that this opinion derives from a confusion: the confusion between the specific form of the Einstein-Hilbert action and the modification of the notions of space and time engendered by GR. The Einstein-Hilbert action might very well be a low energy approximation of something else. But the modification of the notions of space and time has to do with the diffeomorphism invariance and the background independence of the action, not with its specific form. If we make this confusion, we underestimate the novelty of the physical content of GR. The challenge of quantum gravity is precisely to fully incorporate this radical novelty into QFT. In other words, the task is to understand what is a general relativistic QFT, or a background independent QFT.

Today many physicists prefer disregarding or postponing these foundational issues and, instead, develop and adjust current theories. The most popular strategy towards quantum gravity, in particular, is to pursue the line of research grown in the wake of the success of the standard model of particle physics. The failure of perturbative quantum GR is interpreted as a replay of the failure of Fermi theory.[2] Namely as an indication that we must modify GR at high energies. With the input of the grand-unified-theories, supersymmetry, and Kaluza-Klein theory, the search for a high energy correction of GR free from bad ultraviolet divergences has led to higher derivative theories, supergravity, and finally to string theory.

Sometimes the claim is made that the quantum theory of gravity has already been found and it is string theory. Since this is a book about quantum gravity without strings, I should say a few words about this claim. String theory is based on a physical hypothesis: elementary objects are extended,

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[1] In [20], Gell-Mann says that the main lesson to be learnt by from Einstein is "to 'take very seriously' ideas that work, and see if they can be usefully carried much further than the original proponent suggested".

[2] Fermi theory was an empirically successful but nonrenormalizable theory of the weak interactions, like GR is an empirically successful but nonrenormalizable theory of the gravitational interaction. The solution has been the Glashow-Weinberg-Salam electroweak theory, which corrects Fermi theory at high energy.

rather than particle-like. This hypothesis leads to a very rich unified theory, which contains much phenomenology, including (with suitable inputs) fermions, Yang-Mills fields and gravitons, and is believed by many to be free of ultraviolet divergences. The price to pay for these theoretical results is a gigantic baggage of additional physics: supersymmetry, extra dimensions, an infinite number of fields with arbitrary masses and spins, and so on.

So far, nothing of this new physics shows up in experiments. Supersymmetry, in particular, has been claimed to be on the verge of being discovered for years, but hasn't shown up. Unfortunately so far the theory can accommodate any disappointing experimental result because it is hard to derive precise new quantitative physical predictions, with which the theory could be falsified, from the monumental mathematical apparatus of the theory. Furthermore, even recovering the real world is not easy within the theory: the search for a compactification leading to the standard model, with its families and masses and no instabilities, has not yet succeeded, as far as I know. It is clear that string theory is a very interesting hypothesis, but certainly not an established theory. It is therefore important to pursue alternative directions as well.

String theory is a direct development of the standard model and is deeply rooted in the techniques and the conceptual framework of flat space QFT. As I shall discuss in detail all along this book, many of the tools used in this framework – energy, unitary time evolution, vacuum state, Poincaré invariance, S-matrix, objects moving in a spacetime, Fourier transform ... – do not make sense anymore in the quantum gravitational regime, in which the gravitational field cannot be approximated by a background spacetime – perhaps not even asymptotically.[3] Therefore string theory does not address directly the main challenge of quantum gravity: understanding what is background independent QFT. Facing this challenge directly, before worrying about unification, leads, instead, to the direction of research investigated by loop quantum gravity.[4]

The alternative to the line of research followed by string theory is given by the possibility that the failure of perturbative quantum GR is *not* a replay of Fermi theory. That is, it is not due to a flaw of the GR action, but, instead, it is due to the fact that the conventional weak field quantum perturbation expansion cannot be applied to the gravitational field.

This possibility is strongly supported a posteriori by the results of loop quantum gravity. As we shall see, loop quantum gravity leads to a picture of the short scale structure of spacetime extremely different from that of a smooth background geometry. (There are hints in this direction from string theory calculations as well [23].) Spacetime turns out to have a nonperturbative, quantized, discrete structure at the Planck scale, which is explicitly described by the theory. The ultraviolet divergences may be cured by this structure. The ultraviolet divergences that appear in the perturbation expansion of conventional QFT may be a consequence of the fact that we erroneously replace this discrete Planck scale structure with a smooth background geometry.

If this is physically correct, ultraviolet divergences do not require the heavy machinery of string theory to be cured. On the other hand, the conventional weak field perturbative methods cannot be applied, because we cannot work with a fixed smooth background geometry. We must therefore adapt QFT to the full conceptual novelty of GR, and in particular to the change in the notion of space and time induced by GR. What are these changes? I sketch an answer below, leaving a

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[3] To be sure, the development of string theory has incorporated many aspects of GR, such as curved spacetimes, horizons, black holes and relations between different backgrounds. But this is far from a background independent framework, such as the one realized by GR in the classical context. GR is not about physics on a curved spacetime, or about relations between different backgrounds: it is about the dynamics of spacetime. A background independent fundamental definition of string theory is being actively searched along several directions, but so far the definition of the theory and all calculations rely on background metric spaces.

[4] It has also been repeatedly suggested that loop gravity and string theory might merge, because loop gravity has developed precisely the background independent QFT methods that string theory needs [21]. Also, excitations over a weave (see section 6.7.1) have a natural string structure in loop gravity [22].